Application of Fusion-Fission to the multi-way graph partitioning problem

Charles-Edmond Bichot

Laboratoire d'Optimisation Globale, École Nationale de l'Aviation Civile/Direction
des Services de la Navigation Aérienne, 7 av. Edouard Belin, 31055 Toulouse, France,
`bichot@recherche.enac.fr`,

WWW home page: <http://www.recherche.enac.fr/~bichot>

Abstract. This paper presents an application of the Fusion-Fission method to the multi-way graph partitioning problem. The Fusion-Fission method was first designed to solve the normalized cut partitioning problem. Its application to the multi-way graph partitioning problem is very recent, thus the Fusion-Fission algorithm has not yet been optimized. Then, the Fusion-Fission algorithm is slower than the state-of-the-art graph partitioning packages. The Fusion-Fission algorithm is compared with JOSTLE, METIS, CHACO and PARTY for four partition's cardinal numbers: 2, 4, 8 and 16, and three partition balance tolerances: 1.00, 1.01 and 1.03. Results show that up to two thirds of time are the partitions returned by Fusion-Fission of greater quality than those returned by state-of-the-art graph partitioning packages.

1 Introduction

For ten years, the state-of-the-art method to solve the multi-way graph partitioning problem is the multilevel method. The multilevel method often used a graph growing algorithm for the partitioning task and a Kernighan-Lin type refinement algorithm. This method has been introduced in [HL95b, KK95, AHK97]. It is a very efficient process which is very fast too. It consists in reducing the number of vertices of the graph, which is sometimes very high (more than 100,000 vertices), by coarsening them. Then, a partition of the coarsenest graph (less than 100 vertices) is built, generally with a graph growing algorithm [KK98a]. After that, the vertices of the partition are successively un-coarsened and the partition refined with a Kernighan-Lin algorithm [KL70, FM82] or a helpful set algorithm [DMP95].

Graph partitioning has many applications. The most famous of them are parallel computing, VLSI design and engineering computation. Thus, graph partitioning is an important combinatorial optimization problem. Because of its great number of applications, there are different graph partitioning problems. The aim of this paper is to study the most classical of them, the multi-way graph partitioning problem, also called k -way graph-partitioning problem [KK98c]. The other graph partitioning problems, such as the Normalized-Cut partitioning problem

[SM00,DGK04] or the Ratio-Cut partitioning problems [DGK07], are not presented in this paper.

This paper presents an application of the Fusion-Fission method to the multi-way graph partitioning problem. The Fusion-Fission method was first created to solve the Normalized-Cut graph partitioning problem [Bic06,Bic07]. Because the multi-way graph partitioning problem and the normalized-cut graph partitioning problems are strongly related, it is interesting to evaluate the efficiency of the same method to both problems, as it has been done for the multilevel method [DGK07].

The work presented in this paper is the first adaptation of Fusion-Fission to the multi-way graph partitioning problem. Thus, all components of Fusion-Fission presented in [Bic07] do not appear in this preliminary adaptation. However, partitions found by this adaptation are quite good regarding partitions returned by state-of-the-art graph partitioning packages.

2 Graph partitioning

The multi-way graph partitioning problem consists in finding a partition of the vertices of a graph into parts of the same size, while minimizing the number of edges between parts. It is well-known that the multi-way graph partitioning problem is NP-complete.

The difficulty of the task is to keep the sizes of the parts equal while minimizing the edge-cut. The value which represents the difference between the sizes of the parts is named the balance of the partition. Because a small difference between the sizes of the parts may lead to a lower edge-cut [ST97], lots of results are presented with partitions not perfectly balanced.

Definition 1 (Partition of the vertices of a graph). *Let $G = (V, E)$ be an undirected graph, with V its set of vertices and E its set of edges. A partition of the graph into k parts is a set $P_k = \{V_1, \dots, V_k\}$ of sub-sets of V such that:*

- *No element of P_k is empty.*
- *The union of the elements of P_k is equal to V .*
- *The intersection of any two elements of P_k is empty.*

The number of parts k of the partition P_k is named the cardinal number of the partition.

Assume that the graph G is weighted. For each vertex $v_i \in V$, let $w(v_i)$ be its weight. For each edge $(v_i, v_j) \in E$, let $w(v_i, v_j)$ be its weight. Then, the weight of a set of vertices $V' \subseteq V$ is the sum of the weight of the vertices of V' : $w(V') = \sum_{v \in V'} w(v)$.

Definition 2 (Balance of a partition). *Let $P_k = \{V_1, \dots, V_k\}$ be a partition of a graph $G = (V, E)$ into k parts. The average weight of a part is: $W_{average} =$*

$\frac{w(V)}{k}$. The balance of a partition is defined as the maximum weight of all parts divided by the average weight of a part:

$$balance(P_k) = \frac{\max_{V_i \in P_k} w(V_i)}{W_{average}} = \frac{k}{w(V)} \max_{V_i \in P_k} w(V_i) .$$

The objective function to minimize is the *cut* function. It is defined as the sum of the weight of the edges between the parts. More formally, let V_1 and V_2 be two elements of P_k :

$$cut(V_1, V_2) = \sum_{u \in V_1, v \in V_2} w(u, v) .$$

Then, the *cut* objective function is defined as:

$$cut(P_k) = \sum_{V_i, V_j \in P_k, i < j} cut(V_i, V_j) .$$

The partition which has the lowest *cut* value is the solution of the multi-way graph partitioning problem. However, because of the size of the graph to partition (several thousands of vertices), and because of the combinatorial nature of the problem, the partition with the lowest *cut* value can not be found. Thus, combinatorial optimization methods are used to solve this problem.

3 The Fusion-Fission adaptation to multi-way graph partitioning

Because principles of Fusion-Fission are described in [Bic07], this paper presents only succinctly this method. Fusion-Fission principles are based on nuclear force between nucleons. This force is responsible for binding of protons and neutrons into atomic nuclei. In the nature, the fifty-six particles of an iron nucleus are more tightly bound together than in any other element. Thus, the Fusion-Fission optimization process consists in splitting and merging atoms to create atoms of maximum binding energy. Nucleons of big atoms are merged into atoms with few nucleons.

An analogy with graph partitioning is easy. Let the nucleons be the vertices of the graph and the atoms the parts of the partition. The binding energy between two nucleons is the edge weight between the corresponding vertices. According to the Fusion-Fission process, parts of the partition are successively merged and split. Then, the cardinal number of the partition changes during the process. Resulting atoms of the Fusion-Fission process should be atoms of the same size. Which means that the final partition is perfectly balanced.

To be as close as possible to the process described before, the Fusion-Fission application to multi-way graph partitioning is an iteration process which works as follows: at each step of the process, a new partition $P_{l'}^{t+1}$ is created based on the preceding partition P_l^t . The fission process consists in splitting each part of

the partition $P_{l'}^{t+1}$ into l parts. Because of its efficiency, the multilevel method has been chosen for the splitting. The fusion process consists in merging the $l' * l$ parts into a partition P' of l' parts. The merging can be viewed as graph partitioning problem where the vertices of the graph are the $l' * l$ parts. Thus, a multilevel method has been chosen for the merging too. Then, the partition P' is refined using a Kernighan-Lin type algorithm (KL). The resulting partition is the partition $P_{l'}^{t+1}$. The initial partition, P_k^0 , is provided by the multilevel method.

The algorithm 1 presents the Fusion-Fission application to multi-way graph partitioning. The number of part of the new partition, l' , changes at each iteration. We decided to force it to follow a binomial distribution centered in k . Then, a list of numbers which follow this binomial distribution is constructed at the beginning of the Fusion-Fission algorithm. Then, each iteration starts by selecting a new number of part l' in this list.

The multilevel method and the Kernighan-Lin type algorithm (KL in the algorithm 1) used are those of the pMETIS software and are both described in [KK98a]. The pMETIS software does not refer to the parallel implementation of METIS, but pMETIS is the name given of the recursive bisection software implemented in the serial METIS package.

The particularity of the Fusion-Fission algorithm is to find several partitions of different cardinal numbers. Moreover, for each partition found during the algorithm's iteration, refinement is a four-step process. The partition is first refined for a balance of 1.00, then for a balance of 1.01, and after, for balances of 1.03 and 1.05. This four-step refinement process greatly decrease the computation time of the algorithm. Since the algorithm code is not optimized as much as the multilevel softwares, its computation time is less relevant than partition quality.

4 Comparison with state-of-the-art graph partitioning packages

4.1 Benchmarks graphs

The performance of the Fusion-Fission adaptation to multi-way graph partitioning is evaluated on a wide range of tests graphs arising in different application domains. These tests graphs have been chosen among classical benchmarks in the literature of graph partitioning. Some of these benchmarks have been tested in some recent papers [BGOM03,SWC04,KcR04,DGK07]. These graphs are both vertex and edge unweighted. The characteristics of these graphs are described in table 1.

All of these benchmarks graphs can be downloaded at the University of Greenwich graph partitioning archive (May 2007):
<http://staffweb.cms.gre.ac.uk/~c.walshaw/partition/>

All the experiments in this paper were performed on an Intel Pentium IV 3.0 GHz processor with 1 Go of memory, running a GNU/Linux Debian operating system.

Algorithm 1 Fusion-Fission

```
procedure FUSIONFISSION( $G = (V, E)$ ,  $k$ ,  $n$ ,  $pMETIS$ ,  $KL$ )  
   $l \leftarrow k$   
   $P \leftarrow pMETIS(G, k)$   
   $P_k^0 \leftarrow P = \{P_1, \dots, P_k\}$   
  for  $t = 1$  to  $n$  do  
    choose a new number of parts  $l'$   
     $P_l^t = \{P_1, \dots, P_l\}$   
    for  $j = 1$  to  $l$  do  
       $V_{l'}' \leftarrow pMETIS(P_j, l')$   
       $V' \leftarrow V' \cup V_{l'}'$   
    end for  
    make a graph  $G'$  based on the set of parts  $V'$   
     $P_{l'}^{t+1} \leftarrow pMETIS(G', l')$   
     $P' \leftarrow KL(P)$   
    if  $l' = k$  and  $cut(P') < cut(P)$  then  
       $P \leftarrow P'$   
    end if  
  end for  
  return  $P$   
end procedure
```

Table 1. Benchmark graphs characteristics.

Graph name	Size		Degree			Description (source)
	$ V $	$ E $	min	max	avg	
add20	2395	7462	1	123	6.23	20-bit adder (Motorola)
data	2851	15093	3	17	10.59	
3elt	4720	13722	3	9	5.81	2D nodal graph (NASA/RIACS)
uk	4824	6837	1	3	2.83	2D dual graph
add32	4960	9462	1	31	3.82	32-bit adder (Motorola)
bcsstk33	8738	291583	19	140	66.74	3D stiffness matrix (Boeing)
whitaker3	9800	28989	3	8	5.92	2D nodal graph (NASA/RIACS)
crack	10240	30380	3	9	5.93	2D nodal graph
wing_nodal	10937	75488	5	28	13.80	3D nodal graph
fe_4elt2	11143	32818	3	12	5.89	
vibrobox	12328	165250	8	120	26.81	Sparse matrix
bcsstk29	13992	302748	4	70	43.27	3D stiffness matrix (Boeing)
4elt	15606	45878	3	10	5.88	2D nodal graph (NASA/RIACS)
fe_sphere	16386	49152	4	6	6.00	
cti	16840	48232	3	6	5.73	3D semi-structured matrix
memplus	17758	54196	1	573	6.10	Memory circuit (Motorola)
cs4	22499	43858	2	4	3.90	3D dual graph
bcsstk30	28924	1007284	3	218	69.65	3D stiffness matrix (Boeing)
bcsstk31	35588	572914	1	188	32.20	3D stiffness matrix (Boeing)
bcsstk32	44609	985046	1	215	44.16	3D stiffness matrix (Boeing)
t60k	60005	89440	2	3	2.98	2D dual graph
wing	62032	121544	2	4	3.92	3D dual graph
brack2	62631	366559	3	32	11.71	3D nodal graph (NASA/RIACS)

4.2 Some graph partitioning packages

The quality of the partitions produced by the Fusion-Fission algorithm is compared with those generated on the same computer by several public domain graph partitioning softwares:

- The CHACO software [HL95a]. This software includes multilevel and spectral algorithms. Because it is more efficient than the spectral algorithm, only the multilevel algorithm of CHACO, described in [HL95b], is compared with Fusion-Fission.
- The JOSTLE software [Wal02]. It is based on a multilevel multi-way partitioning algorithm [WC00].
- The METIS package [KK98b]. This package provides both the pMETIS and the kMETIS softwares. kMETIS is a direct multi-way partitioning algorithm [KK98c]. pMETIS uses a recursive bisection algorithm [KK98a].
- The PARTY software [PD98]. This software is based on a multilevel algorithm and a helpful-sets refinement algorithm [DMP95].

From all of this softwares, two have a balance parameter : JOSTLE and CHACO (with `KL_IMBALANCE`). The two others found partitions with a variable balance.

4.3 Comparisons between graph partitioning softwares

To be compared with the other algorithms, the Fusion-Fission algorithm has been limited to 2,000 iterations. Then, its runtime is between one minute and one hour. This computation time is quite long regarding those of graph partitioning packages which is often less than a second. There are some explanations to this deficiency. The Fusion-Fission algorithm has not been optimized. It makes four refinement steps instead of one (see section 3). However, the Fusion-Fission algorithm is not slow in comparison with metaheuristics applied to graph partitioning [BGOM03,SWC04] which have a computation time of several hours to several days.

Tables 2 and 3 present some comparisons between the public graph partitioning packages presented in section 4.2 and the Fusion-Fission algorithm. Four cardinal numbers have been chosen: $k = 2, 4, 8$ and 16 . CHACO naturally finds partition perfectly balanced. Its results are compared with those of JOSTLE and Fusion-Fission for *balance* = 1.00. pMETIS (labeled pM. in tables 2 and 3) finds partitions for a balance number of 1.01, thus it is compared with JOSTLE and Fusion-Fission for this imbalance. kMETIS and PARTY are compared with JOSTLE and Fusion-Fission for *balance* = 1.03. When an algorithm do not find a partition for the given balance, the result is marked not available (N/A in tables 2 and 3).

In Tables 2 and 3, lines heading “Best” summarize the number of times the algorithms found the best partition quality over the 23 graphs of this benchmark, regarding results of the other algorithms for the same balance. Results show that Fusion-Fission outperforms the other algorithms in all cases except for $k = 8$ and

$k = 16$ with a balance of 1.00. In the two last cases, the Fusion-Fission algorithm does as well as the JOSTLE software. The Fusion-Fission algorithm has not been constrained to find perfectly balanced partitions even if it tries to do so. Thus, in a few cases it does not find perfectly balanced partitions. The Fusion-Fission algorithm is particularly good for the two smallest cardinal numbers, $k = 2$ and $k = 4$. It can be noticed that the JOSTLE software does almost as well as the other softwares, except when it is compared with pMETIS for $k = 16$ and $balance = 1.03$.

5 Conclusion

A new multi-way graph partitioning method has been presented in this paper. This method named Fusion-Fission is based on a previous work we made to solve the normalized cut graph partitioning problem [Bic06,Bic07]. The adaptation of Fusion-Fission to the multi-way graph partitioning problem uses the pMETIS multilevel algorithm and its Kernighan-Lin refinement algorithm.

This method has been compared with four state-of-the-art graph partitioning packages: JOSTLE, METIS, CHACO and PARTY. Classical benchmarks have been used. The partitions searched are of cardinal numbers 2, 4, 8 and 16, with a balance of 1.00, 1.01 and 1.03. Results show that up to two thirds of time are the partitions returned by Fusion-Fission of greater quality than those returned with state-of-the-art graph partitioning packages.

Since Fusion-Fission takes much longer time than state-of-the-art graph partitioning packages, it may be difficult to use it for parallel matrix applications. However, it can be advantageously be used for fields where run-time is less of a concern, as VLSI layout or air traffic management problems.

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Table 2. Comparisons between algorithms for cardinals numbers $k = 2$ and $k = 4$.

Graph	<i>balance</i> = 1.00			<i>balance</i> = 1.01			<i>balance</i> = 1.03		
	JOSTLE	CHACO	FF	JOSTLE	pM.	FF	JOSTLE	kM. PARTY	FF
<i>k</i> = 2									
add20	734	740	699	729	725	677	721	788	666
data	241	279	204	241	218	203	241	244	196
3elt	95	92	90	95	108	90	95	129	88
uk	33	34	21	28	23	21	25	41	19
add32	12	28	11	10	21	10	10	50	23
bcsstk33	12621	10224	10175	12616	10205	10175	12409	14655	12071
whitaker3	136	132	127	136	135	126	135	152	132
crack	207	209	186	196	187	186	199	278	222
wingnodal	1739	1828	1790	1741	1820	1748	1724	2054	1782
fe4elt2	130	130	130	130	130	130	130	143	130
vibrobox	11436	10346	11866	11424	12427	11552	11511	18245	11975
bcsstk29	2898	2917	2843	2898	2843	2818	2997	3904	N/A
4elt	151	179	143	151	154	139	157	258	159
fesphere	466	422	386	466	440	386	468	568	386
cti	347	410	334	342	334	318	342	688	366
memplus	6141	6861	5816	6096	6337	5816	6047	6519	7604
cs4	455	421	414	435	414	414	406	613	418
bcsstk30	6456	6447	6407	6599	6458	6345	6599	8297	6696
bcsstk31	4044	4020	2805	4060	3638	2718	4191	4950	N/A
bcsstk32	6764	5507	4782	7027	5672	4747	6888	5527	5520
t60k	108	101	100	107	100	83	98	103	93
wing	956	952	950	908	950	908	896	1562	927
brack2	754	752	738	751	738	714	715	845	947
Best	2	2	21	5	2	21	5	0	1
<i>k</i> = 4									
add20	1238	1357	1292	1229	1292	1211	1255	1387	1202
data	448	433	459	447	480	430	425	505	511
3elt	212	219	212	210	231	210	201	265	243
uk	69	63	61	67	67	55	67	85	62
add32	45	79	36	40	42	33	41	107	62
bcsstk33	22130	26191	23066	22293	23131	22652	21590	25493	22445
whitaker3	417	398	406	403	406	397	406	575	399
crack	442	479	382	431	382	378	413	589	476
wingnodal	4073	3992	3720	4048	4000	3659	4048	4832	N/A
fe4elt2	396	356	359	375	359	351	368	1780	437
vibrobox	21761	21087	20282	22156	21471	19940	21844	36206	N/A
bcsstk29	9833	8831	8826	9122	8826	8692	9122	10851	N/A
4elt	498	405	378	485	406	351	434	425	364
fesphere	825	868	844	818	872	825	806	1103	819
cti	1355	1016	1049	1357	1113	1029	1329	2294	1089
memplus	10696	11532	10596	10550	10559	10436	10470	10640	11406
cs4	1194	1132	1154	1177	1154	1102	1162	1599	N/A
bcsstk30	25825	17013	17443	25865	17685	16816	25438	24151	N/A
bcsstk31	10190	10184	8201	10066	8770	7812	10134	15279	N/A
bcsstk32	14890	14946	12205	14887	12205	11340	14887	16215	13333
t60k	240	290	255	229	255	255	242	279	272
wing	1922	2161	1937	1840	2086	1937	1824	3454	N/A
brack2	3222	3356	3705	3144	3250	3109	2999	4129	N/A
Best	6	6	12	5	0	19	4	0	0

Table 3. Comparisons between algorithms for cardinal numbers $k = 8$ and $k = 16$.

Graph	<i>balance</i> = 1.00			<i>balance</i> = 1.01			<i>balance</i> = 1.03		
	JOSTLE	CHACO	FF	JOSTLE	pMETIS	FF	JOSTLE	kMETIS	PARTY
<i>k</i> = 8									
add20	1853	1881	1907	1894	1907	1907	1836	2130	2018
data	798	763	842	800	842	758	756	N/A	791
3elt	462	393	388	433	388	364	418	527	432
uk	114	130	101	104	101	101	106	168	148
add32	120	139	N/A	105	81	81	106	351	N/A
bcsstk33	36106	41951	40070	36269	40070	35579	35961	44681	39071
whitaker3	716	712	719	710	719	692	706	1047	759
crack	809	806	773	779	773	721	751	1047	808
wingnodal	6070	6152	6070	6033	6070	6070	5965	8335	6284
fe4elt2	713	651	654	688	654	646	681	801	707
vibrobox	30103	33410	28696	31032	28177	26162	30247	43334	N/A
bcsstk29	17391	16887	17534	16742	16555	15181	17234	24525	N/A
4elt	674	701	635	612	635	604	656	827	722
fesphere	1351	1337	1330	1280	1330	1302	1274	1643	1277
cti	2257	1886	N/A	2158	2110	2076	2086	3888	2482
memplus	12866	13956	13110	12684	13110	13110	12540	N/A	13119
cs4	1703	1808	1746	1673	1746	1746	1588	2733	1721
bcsstk30	39271	35647	N/A	38746	36357	36357	38228	41052	48539
bcsstk31	15360	17553	16012	17094	16012	14754	19849	20647	N/A
bcsstk32	29281	25810	23601	26655	23601	23601	25343	39817	N/A
t60k	556	593	561	532	561	561	530	1309	581
wing	3028	3221	3205	2918	3205	3205	2911	5748	N/A
brack2	8007	8061	7844	8037	7844	7844	7757	10171	N/A
Best	9	6	9	7	5	16	8	0	0
<i>k</i> = 16									
add20	2555	2269	2504	2532	2504	2504	2565	N/A	2510
data	1299	1279	1309	1299	1370	1278	1263	4857	1475
3elt	645	641	665	621	665	607	603	969	754
uk	211	211	N/A	190	189	189	180	384	220
add32	269	217	128	239	128	128	180	N/A	N/A
bcsstk33	59884	61800	59791	61505	59791	58694	57553	123044	61630
whitaker3	1172	1241	1237	1138	1237	1180	1147	1436	1277
crack	1245	1277	1255	1212	1255	1197	1191	2296	1263
wingnodal	9083	9327	9290	9091	9290	8962	8947	10097	9006
fe4elt2	1194	1095	1152	1146	1152	1083	1140	1500	1236
vibrobox	35447	42634	37441	36233	37441	36398	34521	N/A	N/A
bcsstk29	28294	26239	N/A	28062	28151	26422	28338	147143	N/A
4elt	1081	1099	1056	1034	1056	1048	1012	5077	1282
fesphere	1918	2061	2030	1759	2030	1952	1741	2495	N/A
cti	3402	3122	3181	3345	3181	3181	3262	4760	N/A
memplus	14510	15654	N/A	15085	14942	14942	13958	15804	14831
cs4	2518	2550	2538	2519	2538	2538	2477	3614	2488
bcsstk30	87824	79046	N/A	87472	77293	77293	81764	93834	N/A
bcsstk31	27897	29364	27180	27954	27180	27180	27388	189562	N/A
bcsstk32	46954	46266	43371	48162	43371	43371	48395	50660	N/A
t60k	977	1027	N/A	969	998	998	984	1347	1097
wing	4761	4806	4666	4623	4666	4666	4681	7712	N/A
brack2	13318	13117	12655	13337	12655	12655	13164	150514	N/A
Best	9	7	8	7	9	16	9	0	0

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